Stochastic equation for conserved growth in a restricted solid-on-solid model

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We apply the master-equation method naturally extended for the nonlocal growth process to directly deriving the continuum stochastic equation for a conserved growth model with a restricted solid-on-solid condition. The Villain-Lai–Das Sarma growth equation we obtain for the model is consistent with the result of recent numerical simulations. Furthermore, we find that only the relaxation of the deposited particles up to the nearest neighborhood for $N=1$ condition or the next-nearest neighborhood for $N>1$ condition determines the scaling property and the universality class of the model, and the higher-neighbor hopping processes play no essential role. The choice of the regularization scheme in the derivation procedure is also discussed.
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I. INTRODUCTION

The study of nonequilibrium surface growth has attracted considerable interest in both theoretical and experimental physics [1–5], and recently more and more attention has been focused on the growth via molecular beam epitaxy (MBE) [6–10]. During the MBE growth process, the conserved growth condition is fulfilled, and evaporation and defects, such as overhangs and vacancies, are considered to be negligible. In recent years, a number of discrete conserved models for MBE have been proposed to describe the kinetic roughening properties of surface growth [1–4,8–10]. By extensively studying these models, one can obtain the scaling behaviors and the corresponding universality classes, and then associate the continuum stochastic equations with the given discrete growth models.

There are two main categories of methods of establishing the correspondence between a continuum growth equation and a discrete model. The first one, which is most used in the research work [8–10], is to numerically simulate the model and compare the obtained scaling exponents with those of the corresponding continuum equation. The other one is to derive the continuum equation from a given discrete model analytically, including the method using the principle of symmetry [11] or reparametrization invariance [12], and the approaches starting from the master equation [13,14]. Among them, a systematic method proposed by Vvedensky *et al.* [14], where the continuum equations can be constructed directly from growth rules of the discrete model based on the master-equation description, has been applied to the derivation of growth equations for some discrete models [14–17], including a solid-on-solid (SOS) model with an Arrhenius-type rate [14], the restricted solid-on-solid (RSOS) model [18], as well as the Wolf-Villain (WV) [8] and Das Sarma–Tamborenea (DT) [9] models. It has been pointed out [19,16] that this method fails for an unrestricted SOS diffusion model with Glauber dynamics [10], but works for the simple relaxation models, such as the RSOS, WV, and DT models.

Recently, a new MBE growth model with a RSOS condition has been proposed and studied by Kim *et al.* [20,21].

This model allows the deposited particle to relax to the nearest site where the RSOS condition on neighboring heights $|\delta h| = 0,1, \ldots, N$ [18] is satisfied, and then has the constraint of the conserved growth condition. Numerical simulations showed that this model follows the Villain-Lai–Das Sarma (VLD) growth equation [6,7]. Since it has been found [15] that the master-equation method of Vvedensky *et al.* [14] works well for the RSOS growth, where the height difference between the nearest-neighbor sites remains restricted, it is interesting to apply the method to this conserved growth model with the RSOS condition both for investigating the essential properties of the growth model and for studying the uncertainty of the method, especially the regularization scheme [16].

In this paper, we analytically study the above conserved model with RSOS condition in $1+1$ dimensions using the procedure of Vvedensky *et al.* [14] and directly derive the VLD growth equation for the model. This result is independent of N and in agreement with recent computer simulations [20,21]. Since, in principle, the deposited particles in this model are possible to hop for a long distance, we naturally extend the procedure of Vvedensky *et al.* for this nonlocal process, while in the previous work of deriving the growth equations [14–17] the relaxation process is restricted to the nearest neighborhood. Moreover, we find that the scaling behavior as well as the universality class of the model are determined by the relaxation process of the particles up to the nearest neighborhood for $N=1$ or up to the next-nearest neighborhood for $N>1$, and the higher-neighbor hopping processes are irrelevant and play a negligible role. In Sec. II, we show the derivation of the continuum equation describing the conserved growth model and consider different choices of the regularization scheme. The discussions are presented in Sec. III, and finally, a conclusion is given in Sec. IV.

II. DERIVATION OF GROWTH EQUATION

The derivation procedure of Vvedensky *et al.* [14] begins with the master-equation description of the microscopic dynamics of the discrete model:

$$\frac{\partial P(\mathbf{H};t)}{\partial t} = \sum_{\mathbf{H}'} W(\mathbf{H}',\mathbf{H})P(\mathbf{H}';t) - \sum_{\mathbf{H}'} W(\mathbf{H},\mathbf{H}')P(\mathbf{H};t),$$

(1)

where $\mathbf{H} = \{h_1, h_2, \ldots\}$ with the column height variable h_i ($i = 1, 2, \ldots$) represents the configuration of the surface, $W(\mathbf{H},\mathbf{H}')$ is the transition rate from the configuration $\mathbf{H}$ to the configuration $\mathbf{H}'$, which reflects the microscopic process of the surface growth, and $P(\mathbf{H};t)$ denotes the joint probability that the surface has the configuration $\mathbf{H}$ at time t.

The above master equation can be turned into the Kramers-Moyal expansion form [22], which reduces to the Fokker-Planck equation

$$\frac{\partial P(\mathbf{H};t)}{\partial t} = -\frac{\partial}{\partial h_i}[K_i^{(1)}P(\mathbf{H};t)] + \frac{1}{2}\frac{\partial^2}{\partial h_i \partial h_j}[K_{ij}^{(2)}P(\mathbf{H};t)],$$

(2)

provided that the system size is large and the intrinsic fluctuations are not too large, as shown by Fox and Keizer [23]. In Eq. (2), the first and second transition moments are defined as

$$K_i^{(1)} = \sum_{\mathbf{H}'} (h_i' - h_i)W(\mathbf{H},\mathbf{H}')$$

(3)

and

$$K_{ij}^{(2)} = \sum_{\mathbf{H}'} (h_i' - h_i)(h_j' - h_j)W(\mathbf{H},\mathbf{H}').$$

(4)

Thus, the equivalent Langevin equation can be obtained:

$$\frac{dh_i}{dt} = K_i^{(1)} + \eta_i(t),$$

(5)

where the Gaussian white noise η_i satisfies

$$\langle \eta_i(t) \rangle = 0, \quad \langle \eta_i(t)\eta_j(t') \rangle = K_{ij}^{(2)}\delta(t-t').$$

(6)

Therefore, with the explicit form of the transition rate $W(\mathbf{H},\mathbf{H}')$ obtained from the dynamical rule of the specific growth model, the nonlinear discrete Langevin equation and the noise covariance can be derived. Then a regularization procedure is used [14,16] to pass to the continuum limit and directly derive the continuum stochastic equation for the discrete model.

In the conserved growth model with the RSOS condition proposed by Kim et al. [20], a particle is deposited onto the substrate randomly and will stay there if the RSOS condition on neighboring heights $|\delta h| = 0, 1, \ldots, N$ is obeyed after deposition. Otherwise, the deposited particle will relax on the surface until it finds the nearest site that satisfies the RSOS condition. This relaxation process is absent in the previous nonconserved RSOS model [18] and produces the constraint of conserved growth. Thus, on the growth surface of the model, the RSOS condition is fulfilled at all sites and the height difference between the nearest-neighbor sites remains not larger than N. Consequently, as pointed out by Park and Kahng [15], the procedure of Vvedensky et al., especially the regularization process, is much more convincing in this model than in other MBE models in which the growth surfaces contain high steps.

In principle, the deposited particle of this conserved model is possible to relax for a long distance, thus this model may contain nonlocal growth processes. Therefore, the form of the transition rate $W(\mathbf{H},\mathbf{H}')$ should include the terms representing the long-distance relaxations, and is written as

$$W(\mathbf{H},\mathbf{H}') = \tau^{-1}\sum_k \left[w_k^{(0)}\delta(h_k', h_k + a)\prod_{j \neq k}\delta(h_j', h_j) + w_k^{(-1)}\delta(h_{k-1}', h_{k-1} + a)\prod_{j \neq k-1}\delta(h_j', h_j) + w_k^{(+1)}\delta(h_{k+1}', h_{k+1} + a) \right.$$

$$\times \prod_{j \neq k+1}\delta(h_j', h_j) + w_k^{(-2)}\delta(h_{k-2}', h_{k-2} + a)\prod_{j \neq k-2}\delta(h_j', h_j) + w_k^{(+2)}\delta(h_{k+2}', h_{k+2} + a)\prod_{j \neq k+2}\delta(h_j', h_j) + \cdots \bigg]$$

$$= \tau^{-1}\sum_k \left\{ w_k^{(0)}\delta(h_k', h_k + a)\prod_{j \neq k}\delta(h_j', h_j) + \sum_{l=1}^{\infty}\left[w_k^{(-l)}\delta(h_{k-l}', h_{k-l} + a)\prod_{j \neq k-l}\delta(h_j', h_j) \right.\right.$$

$$+ w_k^{(+l)}\delta(h_{k+l}', h_{k+l} + a)\prod_{j \neq k+l}\delta(h_j', h_j)\bigg]\bigg\},$$

(7)

where a is the lattice constant, and τ denotes the average deposition time for a layer. The $w_k^{(0)}$ term represents that a particle is deposited at site k where the RSOS condition is obeyed after deposition, and stays there. Thus, this term corresponds to the case of the usual nonconserved RSOS model and is defined as f_k here. For $N = 1$, it is given by [15]

$$f_k \equiv w_k^{(0)} = \Theta(h_{k+1} - h_k)\Theta(h_{k-1} - h_k),$$

(8)

where $\Theta(x)$ is the unit step function defined by

$$\Theta(x) = \begin{cases} 1 & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$

Formula (8) means that the height of site k is not larger than that of nearest-neighbor sites $k+1$ and $k-1$. Otherwise, after a particle is deposited at site k the height difference be-

tween site k and its nearest-neighbor site is larger than 1, which disobeys the $N=1$ RSOS condition. Furthermore, we extend this formula to be more general and include the cases for $N>1$, that is,

$$f_k \equiv w_k^{(0)} = \Theta(h_{k+1} - [h_k - (N-1)a])$$
$$\times \Theta(h_{k-1} - [h_k - (N-1)a]). \quad (9)$$

For the terms $w_k^{(-1)}$ and $w_k^{(+1)}$, site k does not satisfy the RSOS condition f_k, but the nearest-neighbor site $k-1$ or $k+1$ obeys. Then they can be written as

$$w_k^{(-1)} = (1-f_k)f_{k-1}(1-f_{k+1} + \tfrac{1}{2}f_{k+1}) \quad (10)$$

and

$$w_k^{(+1)} = (1-f_k)f_{k+1}(1-f_{k-1} + \tfrac{1}{2}f_{k-1}), \quad (11)$$

respectively, where 1/2 represents that if sites $k-1$ and $k+1$ are equally preferable, one of them is chosen at random. The formulas of $w_k^{(\pm 2)}$, $w_k^{(\pm 3)}$, ... are similar, that is,

$$w_k^{(-2)} = (1-f_k)(1-f_{k-1})(1-f_{k+1})f_{k-2}(1 - \tfrac{1}{2}f_{k+2}),$$
$$w_k^{(+2)} = (1-f_k)(1-f_{k-1})(1-f_{k+1})f_{k+2}(1 - \tfrac{1}{2}f_{k-2}),$$
$$w_k^{(-3)} = (1-f_k)(1-f_{k-1})(1-f_{k+1})(1-f_{k-2})$$
$$\times (1-f_{k+2})f_{k-3}(1 - \tfrac{1}{2}f_{k+3}), \quad (12)$$
$$w_k^{(+3)} = (1-f_k)(1-f_{k-1})(1-f_{k+1})(1-f_{k-2})$$
$$\times (1-f_{k+2})f_{k+3}(1 - \tfrac{1}{2}f_{k-3}),$$

$$. \ . \ . \ .$$

Therefore, the common form of $w_k^{(\pm l)}$ ($l \geq 2$) can be written as

$$w_k^{(\pm l)} = (1-f_k)\left[\prod_{n=1}^{l-1}(1-f_{k-n})(1-f_{k+n})\right]f_{k\pm l}(1 - \tfrac{1}{2}f_{k\mp l}), \quad (13)$$

which describes the situation that sites k, $k\pm 1$, ..., $k\pm(l-1)$ do not obey the RSOS condition, while site $k+l$ or $k-l$ obeys. Then the deposited particle at site k hops to site $k-l$ or $k+l$. From the above formulas (10)–(13), we can obtain the condition

$$w_k^{(0)} + w_k^{(-1)} + w_k^{(+1)} + w_k^{(-2)} + w_k^{(+2)} + \cdots$$
$$= w_k^{(0)} + \sum_{l=1}^{\infty}[w_k^{(-l)} + w_k^{(+l)}] = 1, \quad (14)$$

which guarantees that the average deposition rate per site remains τ^{-1}.

From Eqs. (3) and (4), the first and second transition moments for this conserved growth model become

$$K_i^{(1)} = \frac{a}{\tau}[w_i^{(0)} + w_{i+1}^{(-1)} + w_{i-1}^{(+1)} + w_{i+2}^{(-2)} + w_{i-2}^{(+2)} + \cdots]$$
$$= \frac{a}{\tau}\left[w_i^{(0)} + \sum_{l=1}^{\infty}(w_{i+l}^{(-l)} + w_{i-l}^{(+l)})\right], \quad (15)$$

and

$$K_{ij}^{(2)} = aK_i^{(1)}\delta_{ij}. \quad (16)$$

Thus, using Eqs. (5) and (6), we obtain the discrete Langevin equation for this conserved growth model.

In the next step, we regularize the discrete Langevin equation by expanding the nonanalytic quantities and replacing them with analytic quantities. In this regularization procedure, the step function can be approximated by an analytic shifted hyperbolic tangent function, which is expanded in Taylor series [14–17]. As pointed out by Předota and Kotrla [16], the choice of regularization scheme for the step function has the uncertainty. One expansion form

$$\Theta(x) \approx 1 + \sum_{k=1}^{\infty}A_k x^k \quad (17)$$

was used in some work [14,15,17], where $A_1 > 0$, $A_3 < 0$, $A_5 > 0$, ..., and A_2, A_4, A_6, ... are very small and negligible according to the expansion form of the hyperbolic tangent function [17]. Recently, another choice of the regularization function was considered for some discrete models [16], that is,

$$\Theta(x) \approx \sum_{k=0}^{\infty}A_k x^k, \quad (18)$$

where $1/2 < A_0 < 1$, $A_1 > 0$, and $A_2 < 0$. If $A_0 = 1$, the regularization form (17) is recovered. In the following, we will apply these two choices of regularization to studying the conserved model with the RSOS condition.

First, we use regularization (17) on this discrete model. Taking the limit of the lattice constant $a \to 0$, we expand $(h_j - h_i)$, $j = i\pm 1, i\pm 2, \ldots$, in powers of a, and assume that the discrete height variable of the surface $h_i(t)$ can be replaced by an analytic function $h(x,t)$ with $x = ia$, which is smooth at the macroscopic scale. Therefore, substituting formula (17) into Eqs. (9)–(13), and then into Eqs. (15) and (16), and expanding the height variables, we can obtain the corresponding continuum stochastic equation.

If only the $w^{(0)}$ term in formulas (7) and (15) is considered, we return to the nonconserved RSOS model [18]. Using relation (9), the general form of the probability for $N \geq 1$, and retaining the most relevant terms, we obtain the Kardar-Parisi-Zhang (KPZ) equation [24] for the nonconserved RSOS model

$$\frac{\partial h}{\partial t} = \nu_2 \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + F + \eta, \quad (19)$$

where

$$\nu_2 = \frac{a^3}{\tau}A_1,$$
$$\lambda = \frac{2a^3}{\tau}(-A_1^2 + 2A_2), \quad (20)$$
$$F = \frac{a}{\tau}[1 + 2A_1(N-1)a + (A_1^2 + 2A_2)(N-1)^2 a^2],$$

and the noise covariance is

$$\langle \eta(x,t)\eta(x',t')\rangle = \frac{a^2}{\tau}[1+2A_1(N-1)a]\delta(x-x')\delta(t-t'),$$
(21)

up to $O(a^3)$. It is noted that the coefficients of KPZ nonlinearity $(\nabla h)^2$ and Edwards-Wilkinson (EW) [25] term $\nabla^2 h$ are independent of N, and consistent with that derived in the previous work [15] for $N=1$. Thus, we have analytically verified that N is irrelevant in the nonconserved RSOS model, which has been obtained in the numerical simulations [18].

For the conserved RSOS growth model, the relaxation processes on the growth surface, which are reflected in the $w^{(\pm l)}$ terms of formulas (7) and (15), should be considered. The growth equation we derive is also independent of N. For the transition moments, up to $O(a^5)$ we have

$$K^{(1)}(x) = \frac{a}{\tau}\left[1 - \frac{1}{2}A_1 a^4 \frac{\partial^4 h}{\partial x^4} + \left(\frac{1}{2}A_1^2 - A_2\right)a^4 \frac{\partial^2}{\partial x^2}\left(\frac{\partial h}{\partial x}\right)^2\right]$$
$$+ O(a^6),$$
(22)

and

$$K^{(2)}(x,x') = \frac{a^2}{\tau}\delta(x-x') + O(a^6).$$
(23)

Thus, from Eqs. (5) and (6), we obtain that the continuum stochastic equation for the conserved growth in the RSOS model is the VLD equation

$$\frac{\partial h}{\partial t} = -\nu_4 \nabla^4 h + \lambda_{22}\nabla^2(\nabla h)^2 + F + \eta,$$
(24)

where the coefficients are given by

$$\nu_4 = \frac{a^5}{2\tau}A_1,$$

$$\lambda_{22} = \frac{a^5}{\tau}\left(\frac{1}{2}A_1^2 - A_2\right),$$
(25)

$$F = \frac{a}{\tau},$$

and the noise covariance is written as

$$\langle \eta(x,t)\eta(x',t')\rangle = \frac{a^2}{\tau}\delta(x-x')\delta(t-t').$$
(26)

From the above derivation process, there is no EW term $\nabla^2 h$ in the growth equation. However, if the nonlinearities $\nabla(\nabla h)^{2n+1}$ exist in the higher-order expansion, the EW term would be generated according to the dynamical renormalization-group (DRG) analysis [26,27], and then this model should belong to the EW universality class instead of VLD. Therefore, to study the model more explicitly, we expand $K_i^{(1)}$ to higher orders. We obtain that the terms of $O(a^{2n})$ for $K_i^{(1)}$ [i.e., the odd order of a, $O(a^{2n-1})$, for

$w_i^{(0)} + \Sigma_{l=1}^{\infty}(w_{i+l}^{(-l)} + w_{i-l}^{(+l)})]$ vanish, due to the isotropic growth in the model which contains the symmetry between sites $i+l$ and $i-l$ on the surface. The terms appearing in the stochastic growth equation are (for $m,n \geq 1$) the linear terms $\nabla^{2m+2}h$, the nonlinearities $\nabla^{2m+1}(\nabla h)^{2n+1}$, which generate terms $\nabla^{2m}\nabla^2 h$ according to the DRG analysis of Kshirsagar and Ghaisas [27], and the nonlinearities $\nabla^{2m}(\nabla h)^{2n}$, which lead to terms $\nabla^{2m}(\nabla h)^2$ also according to Kshirsagar and Ghaisas [27]. Thus, the nonlinearities $\nabla(\nabla h)^{2n+1}$ do not arise in the growth equation and consequently no EW term is generated in the model.

Next, we apply the other regularization form (18) to the derivation procedure. It is shown from our deduction that the continuous representation of the first transition moment $K^{(1)}(x)$ is different from formula (22), except for $A_0 = 1$, and then the results for this conserved growth model depend on the regularization scheme chosen. The similar problem has been found by Předota and Kotrla [16] in the study of the WV model.

When we consider all the relaxation processes up to distance l, for $N=1$ the form of the transition moment (15) up to the third order is given by

$$F + \frac{a}{\tau}\left\{(2l+1)(1-A_0^2)^{2l}a^2\left[A_0 A_1 \frac{\partial^2 h}{\partial x^2}\right.\right.$$
$$\left.\left. + (-A_1^2 + 2A_0 A_2)\left(\frac{\partial h}{\partial x}\right)^2\right]\right\},$$

where F represents the average deposition flux. Since in the derivation procedure of this growth model the relaxation to all distances should be included in principle to guarantee the conserved condition, we have $l\rightarrow\infty$. Consequently, the coefficients of the EW and KPZ terms shown above tend to zero, as in the regularization form (18) $A_0 \in (1/2,1)$ and then $1-A_0^2 < 1$. However, the coefficients of the nonlinearities $\nabla(\nabla h)^{2n+1}$ appearing in the higher-order expansion cannot vanish as $l\rightarrow\infty$. For instance, the coefficient λ_{13} of $\nabla(\nabla h)^3$ has the form

$$\frac{a^5}{\tau}\{2l(2l+1)(1-A_0^2)^{2l-1}A_0 A_1^3 - (2l+1)$$
$$\times[4lA_0^2(1-A_0^2)^{2l-1} + (1-A_0^2)^{2l}]A_1 A_2$$
$$+ 3(2l+1)(1-A_0^2)^{2l}A_0 A_3\}.$$

The last term tends to zero when $l\rightarrow\infty$, but the other two do not. Thus, setting $l\rightarrow\infty$ we can obtain

$$\lambda_{13} = \frac{2a^5(A_0 A_1^3 - 2A_0^2 A_1 A_2)}{-3\tau\ln(1-A_0^2)}.$$
(27)

Since $A_0 > 0$, $A_1 > 0$, $A_2 < 0$, and $1-A_0^2 < 0$ for regularization (18), we have $\lambda_{13} > 0$. Therefore, the $\nabla(\nabla h)^3$ nonlinearity, which is the most relevant here, leads to the EW universality class according to both the direct numerical integration study [28] and the DRG analysis [26]. However, this result contradicts recent numerical simulations by Kim et al. [20,21], where no EW behavior was observed.

It is noted that regularization (18) was introduced originally for some discrete models in which the δ function is to be defined and the situations where the argument of the step function is zero have to be distinguished [16]. Since this problem does not exist in the studying of the nonconserved RSOS model as well as the conserved RSOS growth here, the regularization form (17) can be used. From the above results derived and the comparison with recent computer simulations, one can find that the regularization relation (17) is expected to be the preferable choice for this conserved model, while for regularization (18), the result in contradiction with the numerical simulations is obtained. Thus, the discussions below are based on the results of regularization (17).

III. DISCUSSION

According to the above derivation using the regularization form (17), we can find that the terms appearing in the growth equation can be classified as the linear terms $\nabla^{2m+2}h$ and the nonlinearities $\nabla^{2m}(\nabla h)^2$, while the other terms can lead to one of them using the DRG analysis. Among them, the most relevant term is VLD nonlinearity $\nabla^2(\nabla h)^2$ from the renormalization-group viewpoint. Therefore, keeping this nonlinearity as well as the meaningful lowest-order term $\nabla^4 h$, we have the conclusion that the conserved growth model with the RSOS condition is governed by the VLD equation (24), which is consistent with the result of numerical simulations [20,21]. Moreover, from the expression (25) of the coefficients in Eq. (24) and the signs of A_k for regularization (17) shown above, that is, $A_1 > 0$ and A_2 is very small, we have $\nu_4 > 0$, which is in agreement with the phenomenological consideration, and $\lambda_{22} > 0$, which confirms the previous result [20,21] obtained from the argument that in this model surface current is generated from the higher sloped region to the lower sloped region.

It is noted that the terms in VLD equation (24), which determine the scaling property and the universality class of the model, come from the expansion of the transition moment $K_i^{(1)}$ up to $O(a^5)$ [or $O(a^4)$ for $w_i^{(0)}$ and $w_{i\pm l}^{(\mp l)}$], and then the higher-order expansion is irrelevant and has no essential influence. Thus, it is interesting to estimate the lowest order of the expansion of $w_i^{(0)}$ and $w_{i\pm l}^{(\mp l)}$, which correspond to different relaxation processes of the deposited particles. From formula (9) and regularization form (17) we obtain that the lowest order in the expansion of $w_i^{(0)}$ is $O(1)$, and that of $w_{i+1}^{(-1)}$ and $w_{i-1}^{(+1)}$ is $O(a^2)$ for $N=1$ and $O(a)$ for $N>1$ from Eqs. (10) and (11). Moreover, it is shown from formulas (12) and (13) that the lowest order of $w_{i\pm2}^{(\mp2)}$ is $O(a^6)$ for $N=1$ and $O(a^3)$ for $N>1$, the lowest order of $w_{i\pm3}^{(\mp3)}$ is $O(a^{10})$ for $N=1$ and $O(a^5)$ for $N>1$, ..., and commonly, the lowest order in the expansion of $w_{i\pm l}^{(\mp l)}$ is $O(a^{2(2l-1)})$ for $N=1$ and $O(a^{2l-1})$ for $N>1$. Therefore, when $l \geq 2$ for $N=1$, which corresponds to the relaxation process beyond the nearest neighborhood, or $l \geq 3$ for $N>1$, which corresponds to the relaxations beyond the next-nearest neighborhood, the expansion of $w_{i\pm l}^{(\mp l)}$ is higher than the meaningful $O(a^4)$, and then cannot have relevant influence on the scaling property of the growth model. Recent computer simulations [21] have shown that the probability for a deposited particle to hop for a long distance is very low, here we can have a further conclusion from the above analytic study that only the relaxation process of the particle up to the nearest neighborhood for $N=1$ or the next-nearest neighborhood for $N>1$ determines the scaling behavior and the universality class of the model, and the higher-neighbor relaxation processes are irrelevant and play an inessential and negligible role. Thus, essentially this conserved model is still a local model.

Both the above analytic investigation and recent numerical simulations have verified that there is no EW term in this conserved model. The physical origin of this result can be obtained by studying the microscopic process of the growth. Kim et al. [20,21] have argued that since a deposited particle is allowed to hop equally in both up and down directions in the model, there is no net surface current and consequently no EW term can be generated according to the method of the tilt-dependent surface current analysis proposed by Krug, Plischke, and Siegert [29]. However, for this conserved model with $N=1$ RSOS condition, the uphill hopping of the deposited particle in $1+1$ dimensions can occur only beyond the nearest neighborhood, which has been shown above to be irrelevant and have an inessential influence on the scaling behavior and the universality class of the model. For the relevant nearest-neighbor relaxation process in $1+1$ dimensions, a particle is not allowed to hop to the higher nearest-neighbor site, otherwise the RSOS condition $|\delta h| \leq 1$ cannot be satisfied there. Therefore, the argument of Kim et al. is questionable for $N=1$ and $1+1$ dimensions, while for $N>1$ the above problem is not encountered and the argument of Kim et al. is reasonable.

IV. CONCLUSION

We have directly derived the VLD equation for a conserved growth model with the RSOS condition using the master-equation method of Vvedensky et al., which can be naturally extended for the nonlocal growth process. The growth equation as well as the coefficients are verified to be independent of N. Although the model can contain the long-distance hopping processes in principle, the relevant relaxation that determines the scaling behavior and the universality class is only up to nearest neighborhood for $N=1$ or up to next-nearest neighborhood for $N>1$, and the higher-neighbor hoppings are irrelevant and play no essential role. These results are based on the regularization form (17) which is expected to be preferable for this conserved RSOS model, while for the other regularization scheme (18), the results we derive are in contradiction with recent numerical simulations. Thus, for this conserved RSOS model the results of the master-equation method depend on the regularization scheme used. To choose the proper regularization, the specific growth rules of the discrete model as well as the comparison with the numerical simulations should be considered.

ACKNOWLEDGMENT

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